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High-Dielectric-Constant Materials as High-Frequency Capacitors

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Abstract

In the design of high-frequency (or fast-impulse) capacitors required to hold off high voltages, one can use high-dielectric-constant materials (e.g., certain ceramics). This paper considers some of the high-frequency problems of such capacitors and techniques to mitigate them.

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1. Introduction

In some kinds of high-power electromagnetic guiding structures blocking capacitors are used to keep long-time-duration electric fields at low values on parts of the structure. There is a need, however, for these blocking capacitors to act more as short circuits for fast pulses with correspondingly high frequencies. Just how low an impedance such capacitors can represent is then an important design consideration.

There are various ways to make capacitors, such as alternating sheets of metal and dielectric, including high-frequency designs [4, 5]. Another type consists of blocks of high-dielectric-constant dielectrics (e.g., certain ceramics) in various shapes [2]. With relative dielectric constants of a few thousand, these should behave like metals at appropriate high frequencies. Here we give some approximate analysis to quantify this.

2. Circularly Cylindrical Rod

As illustrated in Fig. 2.1 let there be a dielectric rod of radius a and length ℓ. The length is not significant in the analysis since this is treated on a per-unit-length basis. Its constitutive parameters are

$$\begin{aligned} \mu &= \mu_0 \text{ free space permeability} \\ \varepsilon &= \varepsilon_2 >> \varepsilon_0 \text{ permittivity} \\ \sigma &= 0 \text{ conductivity} \end{aligned} \tag{2.1}$$

The external medium has (appropriate to gas or oil).

$$\begin{aligned} \mu &= \mu_0 \\ \varepsilon &= \varepsilon_2 >> \varepsilon_0 \\ \sigma &= 0 \end{aligned} \tag{2.2}$$

For convenience we have the relative dielectric constant

$$\varepsilon_r = \frac{\varepsilon_2}{\varepsilon_1} >> 1 \tag{2.3}$$

For present purposes we can use ε_0 in the above.

Treating the rod as an electromagnetic boundary value problem, the incident wave can have various configurations. An interesting case has a TEM wave propagating along one of the metal rods when it encounters the dielectric rod and propagates along it to reach a second metal rod. Another metal conductor (not shown) serves as the return path for the TEM wave. While serving as a blocking capacitor at low frequencies we would like it to behave as a metal rod at high frequencies. Of course, it is not a metal rod but only approximates one.

While the wave may be propagating at a high speed (say c) in the external medium, it is propagating at a much slower speed, $\varepsilon_r^{-1/2} c << c$, in the dielectric. As such it is not propagating significantly in the z direction, but is propagating approximately radially in the dielectric rod. For this analysis we have cylindrical coordinates (Ψ, ϕ, z) as

$$x = \Psi \cos(\phi) \quad , \quad y = \Psi \sin(\phi) \tag{2.4}$$

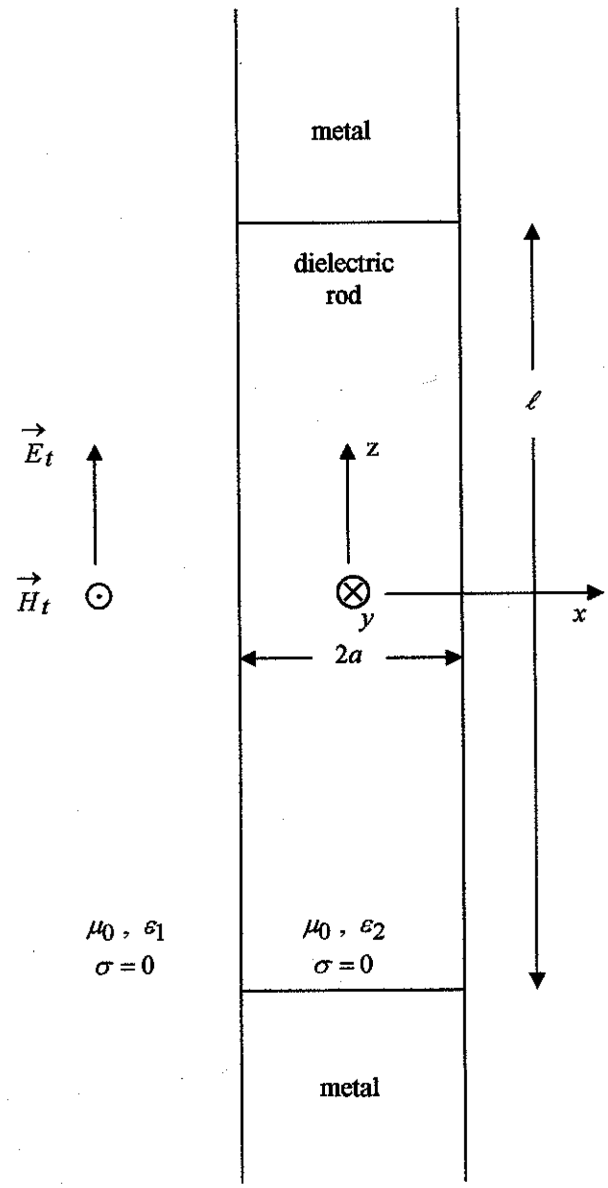

Fig. 2.1. Circularly Cylindrical Rod Capacitor

with the boundary at $\Psi = a$.

Neglecting the z variation we have in the dielectric (similar to the analysis in [1])

$$
\begin{aligned}
E_z &= E_0 \; a_0 \; J_0(k_2\Psi) \\
H_\phi &= -j\frac{E_0}{Z_2} \; J_0'(k_2\Psi) \\
k_2 &= \omega\left[\mu_0\varepsilon_2\right]^{1/2} = \frac{\omega}{c}\varepsilon_r^{1/2} = \varepsilon_r^{1/2} \; k_0 \\
Z_2 &= \left[\frac{\mu_0}{\varepsilon_2}\right]^{1/2} = \varepsilon_2^{-1/2} \; Z_0 \\
k_0 &= \omega\left[\mu_0\varepsilon_0\right]^{1/2} = \frac{\omega}{c} \;, \quad Z_0 = \left[\frac{\mu_0}{\varepsilon_2}\right]^{1/2}
\end{aligned}
\tag{2.5}
$$

We then have an impedance per unit length at $\Psi = a$ of

$$
\begin{aligned}
\tilde{Z}'(j\omega) &= \frac{1}{2\pi a}\frac{E_z}{H_\phi} = \frac{jZ_2}{2\pi a}\frac{J_0(k_2a)}{J_0'(k_2a)} \\
&= -\frac{jZ_2}{2\pi a}\frac{J_0(k_2a)}{J_1(k_2a)}
\end{aligned}
\tag{2.6}
$$

For small arguments the Bessel functions give

$$
\begin{aligned}
J_0(k_2a) &= 1 + O\left([k_2a]^2\right) \\
J_1(k_2a) &= \frac{k_2\Psi}{2}\left[1 + O\left([k_2a]^2\right)\right] \\
\tilde{Z}'(j\omega) &= \frac{-jZ_2}{\pi a^2 k_2}\left[1 + O\left([k_2a]^2\right)\right] \\
&= \frac{-j}{\omega\pi a^2\varepsilon_2}\left[1 + O\left([k_2a]^2\right)\right] \\
&= \frac{-jS'}{\omega}\left[1 + O\left([k_2a]^2\right)\right]
\end{aligned}
\tag{2.7}
$$

$S' = \left[\pi a^2\varepsilon_2\right]^{-1} \equiv$ susceptance (reciprocal capacitance) per unit length

This verifies the well-known capacitance at low frequencies.

As frequency is increased the behavior deviates from that of a simple capacitor. The impedance per unit length tends to zero at the first zero of J_0, i.e.,

$$k_2 a = \omega a\left[\mu_0 \varepsilon_2\right]^{1/2} = \omega \frac{a}{c} \varepsilon_r^{1/2} \simeq 2.4 \tag{2.8}$$

Further increasing the frequency changes the sign of $\tilde{Z}'$ (becoming inductive). The magnitude increases, going to ∞ at the first zero of J_1, i.e.,

$$k_2 a \simeq 3.8 \tag{2.9}$$

For a simple example one might have

$$\begin{aligned} a &= 1 \text{ cm} \\ \varepsilon_r &= 2000 \\ \mathrm{S}' &\simeq 1.8 \times 10^{11}\ \mathrm{F}^{-1}\ \mathrm{m}^{-1} \\ \mathrm{C} &= \left[\mathrm{S}'\ell\right]^{-1} \simeq 56 \text{ pF} \\ \mathrm{f} &= \frac{\omega}{2\pi} = k_2 a \frac{c}{2\pi a} \varepsilon_r^{-1/2} \simeq 0.26 \text{ GHz at first } \tilde{\mathrm{Z}}' \text{ zero} \end{aligned} \tag{2.10}$$

Note that dielectric losses at high frequency have been neglected. Such would make k_2 complex and avoid the high-impedance problem at $k_2 a \simeq 3.8$. Nevertheless, this analysis indicates a possible high-frequency problem.

3. Array of Rods

In order to improve the high-frequency performance of the rod capacitor, the radius can be decreased, but at the expense of lowering the capacitance. An alternate approach is to use some number N_c of rods retaining the total cross section πa^2 by reducing the radius of each rod to $a N_c^{-1/2}$, thereby raising the characteristic frequencies by a factor $N_c^{1/2}$.

As indicated in Fig. 3.1, let us place the rod-capacitor center lines on a circular cylinder of radius b with equal spacing in angle $\phi_c = 2\pi / N_c$. The array of rods is terminated at both ends in metal with radius of order b or greater. Restrict $k_1 b << 1$ so that we can neglect the impedance per unit length contribution by the dielectric medium ε_1 (other than as a simple capacitive contribution). This is possible due to the large values of ε_r, wavelengths in the ε_1 dielectric being much larger than those in the ε_2 dielectric rods.

The above considerations are for the case that equal displacement currents are desired in each arm, corresponding to a uniform current distribution around circularly cylindrical rods terminating the rod array at both ends. If this distribution should not be uniform, but like that for a circularly cylindrical conductor parallel to a ground plane, then one can vary the angles separating the various rods. Considerations in a previous paper [3] can be used to optimize the distribution of the rods.

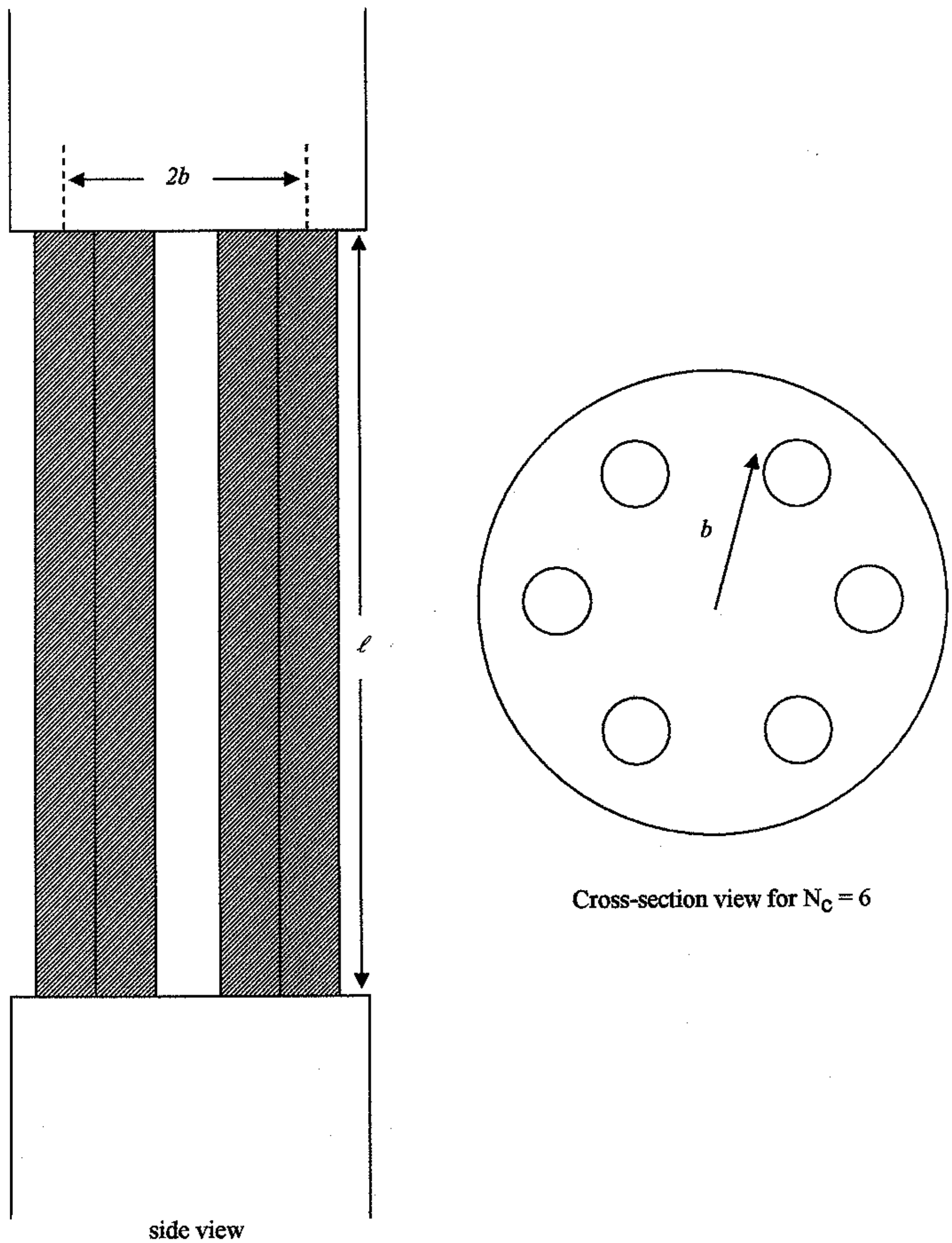

Fig. 3.1. Array of Rod Capacitors: N_c Capacitors in Circular Formation.

4. Circularly Cylindrical Shell

Carrying the argument further let us now distribute the ε_2 dielectric in a circularly cylindrical shell (a tube) as illustrated in Fig. 4.1. Letting

$$\begin{aligned} \Psi_1 &\equiv \text{inner radius} \\ \Psi_2 &\equiv \text{outer radius} \\ \Delta &\equiv \Psi_2 - \Psi_1 = \text{thickness} \\ b &\equiv \frac{1}{2}[\Psi_2 + \Psi_1] = \text{average radius} \end{aligned} \tag{4.1}$$

we can compute $\tilde{Z}'$ for this configuration.

One can perform the calculation in cylindrical coordinates and obtain Bessel-Function formulae. For present purposes we can estimate the parameters for $\Delta << b$, neglecting the displacement current for radius $\Psi < \Psi_1$. This gives

$$\tilde{Z}'(j\omega) = \frac{Z_2}{2\pi b}\frac{1 + e^{-j2k_2\Delta}}{1 - e^{-j2k_2\Delta}} = -j\frac{Z_2}{2\pi b}\cot(k_2\Delta) \tag{4.3}$$

For low frequencies this gives

$$\begin{aligned} \tilde{Z}'(j\omega) &= -j\frac{Z_2}{2\pi b}\frac{1}{k_2\Delta}\left[1 + O\left([k_2\Delta]^2\right)\right] \\ &= -\frac{j}{\omega 2\pi b\Delta\varepsilon_2}\left[1 + O\left([k_2\Delta]^2\right)\right] \\ &= -\frac{jS'}{\omega}\left[1 + O\left([k_2\Delta]^2\right)\right] \\ S' &= [2\pi b\Delta\varepsilon_2]^{-1} \end{aligned} \tag{4.4}$$

For comparison to the single rod we have the same capacitance. So we equate the cross-section areas as

$$\begin{aligned} 2\pi b\Delta &= \pi a^2 \\ b\Delta &= \frac{a^2}{2} \end{aligned} \tag{4.4}$$

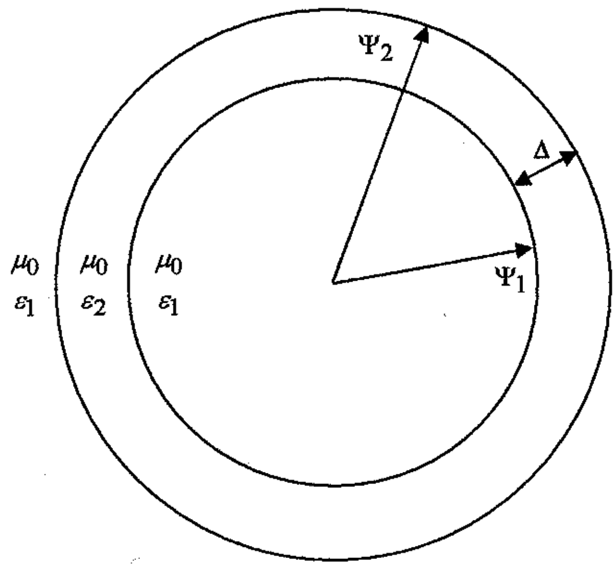

Fig. 4.1. Cylindrical Shell Capacitor

Making b large makes Δ small for a given cross-section area. One can also reduce ℓ to reduce the cross-section area for a given capacitance. However, the minimum ℓ may be constrained by high-voltage-standoff considerations.

As ω is increased, $\tilde{Z}'$ has a zero at

$$k_2\Delta = \omega\Delta\left[\mu_0\varepsilon_2\right]^{1/2} = \omega\frac{\Delta}{c}\varepsilon_r^{1/2} = \frac{\pi}{2} \tag{4.6}$$

Further increasing the frequency there is a pole at

$$k_2\Delta = \pi \tag{4.7}$$

For a simple example, keep the cross-section area the same as in (2.10)

$$2\pi b\Delta = \pi^2 \tag{4.8}$$

but let

$$b = 3\text{ cm} \quad , \quad \Delta = \frac{a^2}{2b} \simeq 1.7\text{ mm} \tag{4.9}$$

The first $\tilde{Z}'$ zero is then at

$$\text{f} = \frac{\omega}{2\pi} = \frac{k_2\Delta}{2\pi}\frac{c}{\Delta}\varepsilon_r^{-1/2} = \frac{c}{4\Delta}\varepsilon_r^{-1/2} \simeq 1\text{ GHz} \tag{4.10}$$

This is ideally corrected to account for the space inside the tube, especially at the higher frequencies.

5. Concluding Remarks

There are then some high=frequency effects with which one needs to be concerned. These can be pushed to higher frequencies by the previously discussed techniques of multiple rods, a cylindrical shell, or a combination of these. Other cross-section shapes (such as star or asterisk shaped) can also be considered to give short paths (like Δ) for fields to propagate into the dielectric. For the analysis, a frequency-independent real dielectric constant has been assumed. Such capacitive materials may also have some high-frequency losses which will mitigate the problem to some degree.

References

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